647510 **BSCbooCC5030** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
December -2020 (OLD COURSE)
BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1(CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1(A) Answer any TWO. (10)

(1) Draw Hasse diagram of

(a) (S_{60}, D) (b) $(P(X), \subseteq)$, where $X = \{a, b, c, d\}$.

(2) State and prove absorption laws for a lattice.

(3) Prove that direct product of two lattices is a lattice.

Q.1(B) Attempt any ONE. (04)

(1) State and prove modular inequality.

(2) Prove that every chain is a distributive lattice.

Q.2(A) Answer any TWO. (10)

(1) State and prove De Morgan's laws for a Boolean algebra.

(2) Let $(B, *, \oplus, ', 0, 1)$ be a Boolean algebra and $x_1, x_2 \in B$. In usual notations, prove that $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$.

(3) (i) Express the Boolean expression $\alpha(x_1, x_2, x_3) = x_1 \oplus x_2$ in sum of product canonical form.

(ii) Express the Boolean expression $x_1 * x_2$ in product of sum form.

Q.2(B) Attempt any ONE. (04)

(1) Let $(B, *, \oplus, ', 0, 1)$ be a Boolean algebra. If a and b be two distinct atoms of B then prove that $a * b = 0$.

(2) (i) Reduce the expression $\alpha(x, y, z) = x'yz + xyz + x'yz' + xyz'$ using Karnaugh map.

(ii) Find cube array representation of the expression $f(x, y, z) = x'y + xy$.

Q.3(A) Answer any TWO. (10)

(1) Show that the function $f(z) = |z|^2$ is not differentiable at any point except origin.

(2) State and prove necessary condition for a function to be analytic.

(3) Show that the function $u(x, y) = y^3 - 3x^3y$ is harmonic. Find harmonic conjugate v of u and also determine the analytic function $f(z) = u + iv$.

Q.3(B) Attempt any ONE. (04)

(1) Prove that the following functions are entire.

(i) $\cos z$ (ii) $r^2 (\cos 2\theta + i \sin 2\theta)$

(2) Show that the function

$$f(z) = \begin{cases} \dfrac{\bar{z}^2}{z}\ ; & z \quad 0 \\ 0; & z = 0 \end{cases}$$

is not analytic at origin although Cauchy Reimann equations are satisfied at origin.

Q.4(A) Answer any TWO. (10)

(1) Evaluate $\displaystyle\int_0^{2+i} (\bar{z})^2 \, dz$

(i) along the real axis to 2 and then vertically to 2 + i.

(ii) along the line $2y = x$.

(2) Derive Cauchy-Reimann equations in polar form.

(3) State and prove Cauchy's integral formula.

Q.4(B) Attempt any ONE. (04)

(1) If C_R denotes the upper half of the circle $|z| = r$ $(r > 2)$ taken in counter clockwise direction then show that

$$\left| \int_{C_R} \left(\frac{2z^2 - 1}{z^4 + 5z^2 + 4} \right) dz \right| \quad \frac{r(2r^2 + 1)}{(r^2 - 1)\,(r^2 - 4)}$$

(2) Evaluate $\int_C \left(\frac{z}{(z-1)(z-2)} \right) dz,$ were C is

(i) $|z| = 1/2$ (ii) $|z| = 3/2$.

Q.5(A) Answer any TWO. (10)

(1) Evaluate $\int_C \left(\frac{1}{(z^2+4)^2} \right) dz,$ were C is $\mathit{the\ circle}\ |z - i| = 2.$

(2) State and prove Morera's theorem.

(3) State and prove fundamental theorem of algebra.

Q.5(B) Attempt any ONE. (04)

(1) State and prove Cauchy's inequality.

(2) State and prove Liouville's theorem.

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